Optimal facility location and equipment selection for whey reuse

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Overview

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Outline

Why Optimize Whey Supply Chain?

- A potentially important source of nutrients
- Disposing of whey in the environment is wasteful and pollutant
- High value-added products with applications in:
  - Dairy,
  - Meat,
  - Dry foods,
  - Bakery,
  - Confectionery,
  - Food supplements,
  - Drinks,
  - Pharmaceuticals,
  - ...

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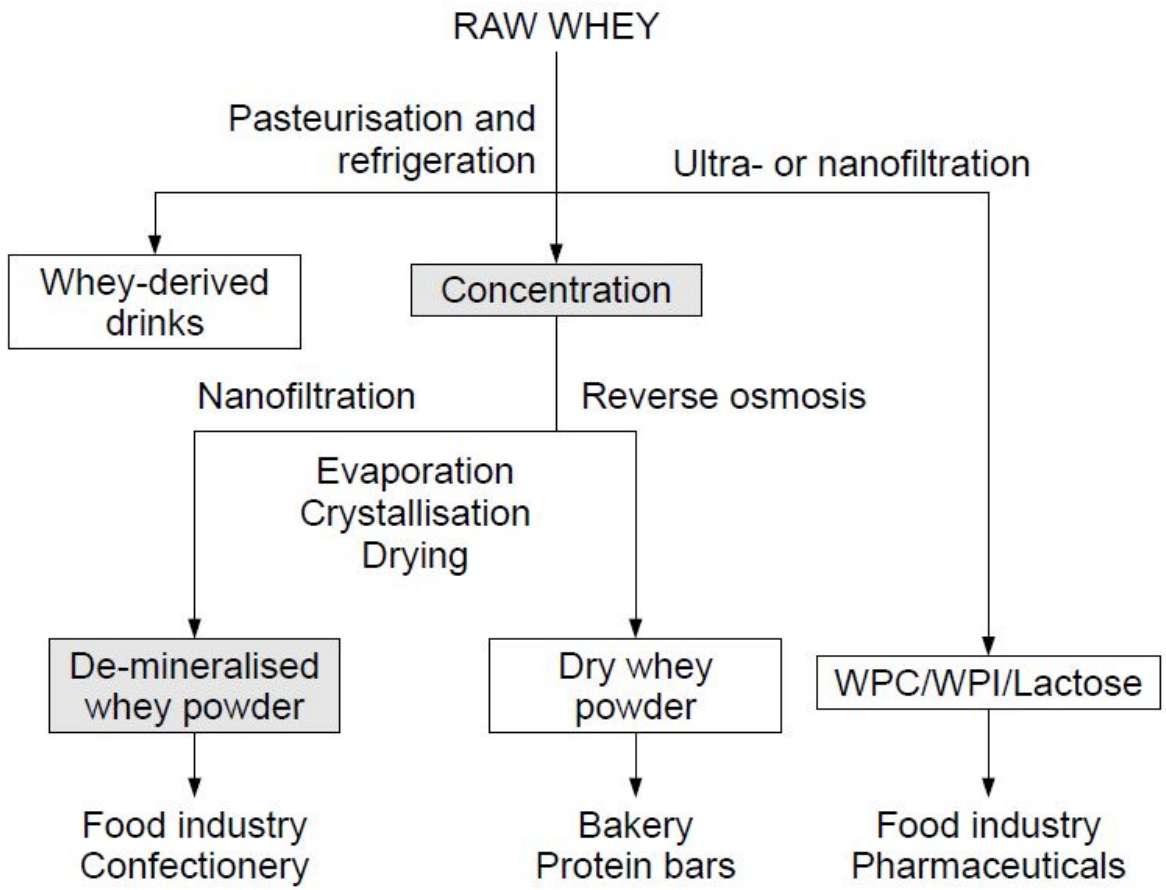
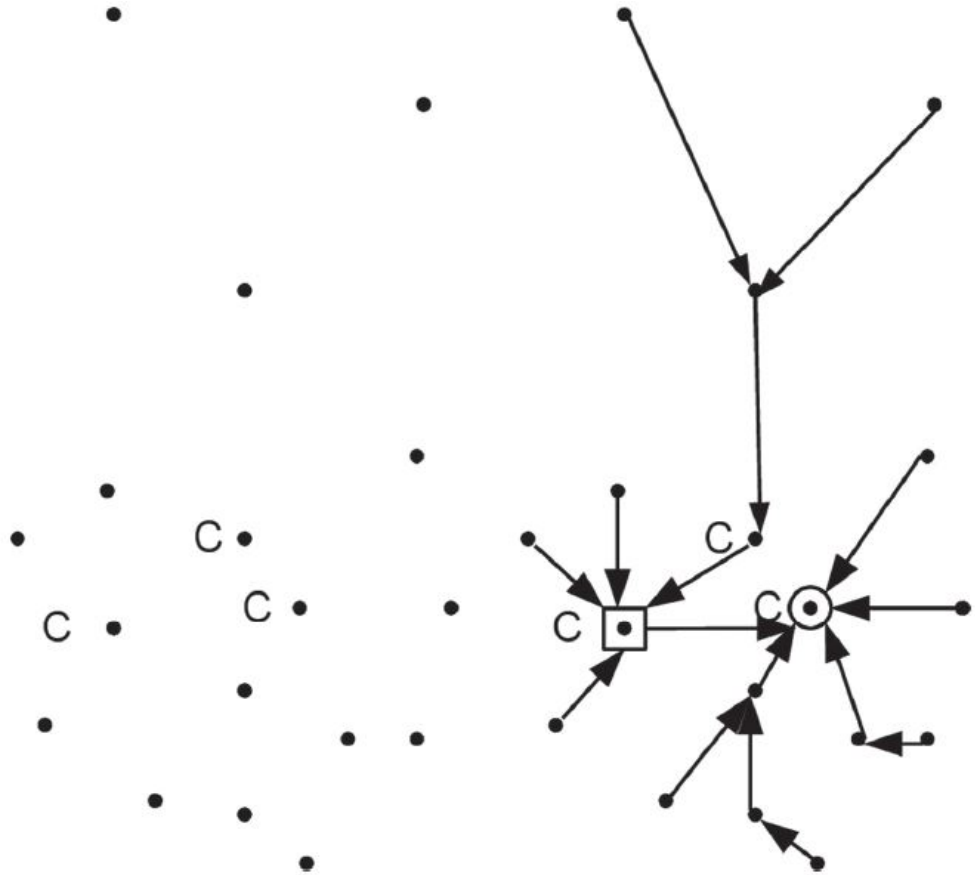
Possible processing paths

- 40% demineralized whey powder (40DWP)



Whey supply chain

- Candidates (marked with a C)
- Collection centers (square)
- Plants (circle)
- Hierarchical system



(a) Before design (b) After design

Outline

Definitions

Service

A **service** is a set of tasks that collection centers or plants perform.

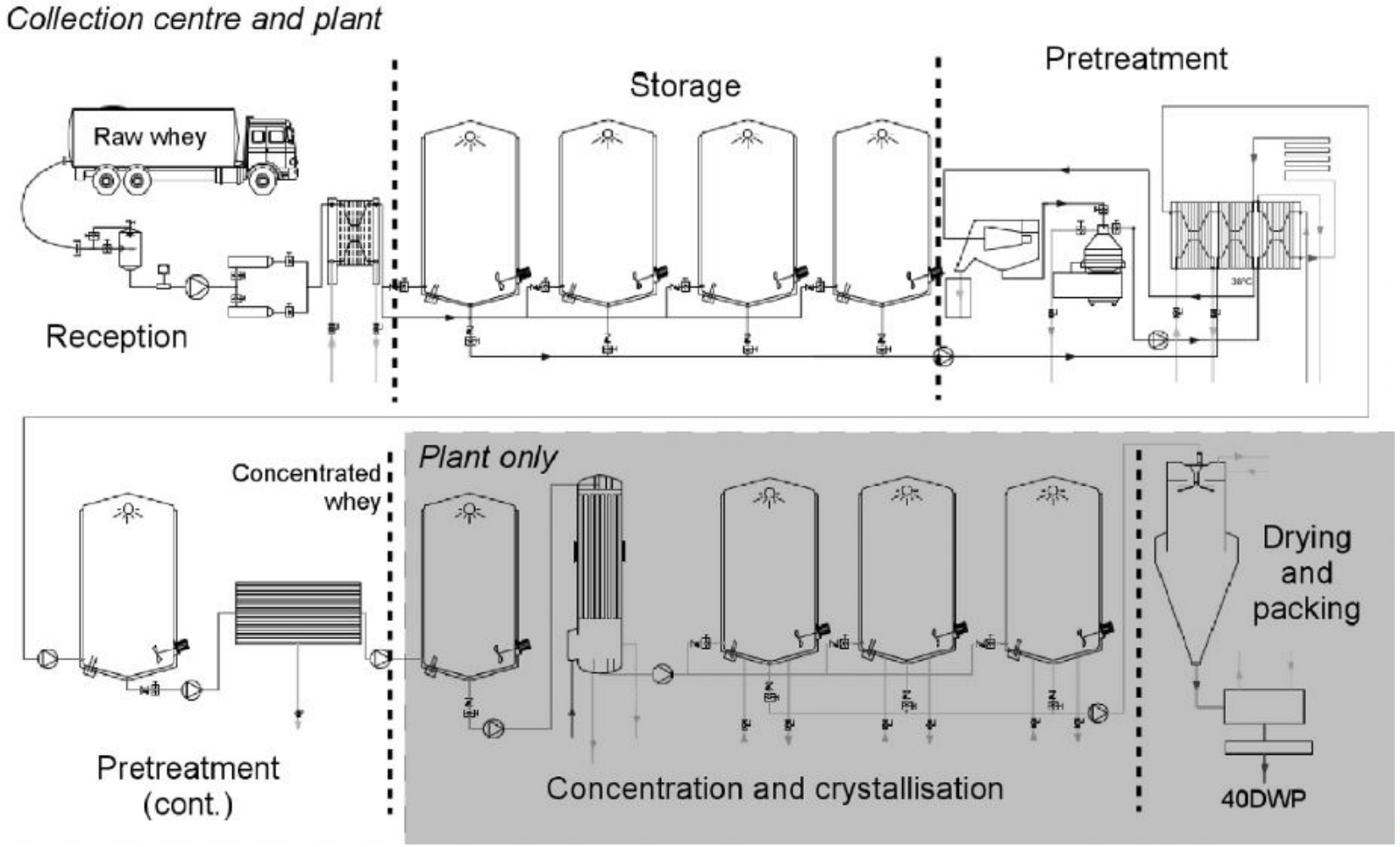
Stage

Each service consists of an ordered set of **stages** that form that service.

Configuration

A valid combination of machines that perform stage operations (tasks) of a service is called a **configuration** of that service.

40DWP process diagram



Sets and Indexes

Notation	Definition
n	The number of nodes (cheese makers)
N	Set of nodes, i.e., $N = \{1, 2, \ldots, n\}$
V	Set of candidate centers $(V \subseteq N)$
i, j	Indexes for the nodes
H_i^c	Set of all configurations available for a collection center located in $i \in V$
H_i^p	Set of all configurations available for a plant located in $i \in V$
h	Index for the configurations

Continuous Variables

We have three types of flows

1. $x_{ij}^{rc} :=$
   Raw whey transported from cheese maker $i \in N$ to collection center $j \in V$
2. $x_{ij}^{rp} :=$
   Raw whey transported from cheese maker $i \in N$ to plant $j \in V$
3. $x_{ij}^{cp} :=$
   Concentrated whey transported from collection center $i \in V$ to plant $j \in V$

Continuous Variables

We have three types of production

1. $y_{ih}^{rc} :=$
   Concentrated whey produced from raw whey in collection center $i \in V$ by configuration $h \in H_i^c$
2. $y_{ih}^{rp} :=$
   40DWP produced from raw whey in plant $i \in V$ by configuration $h \in H_i^p$
3. $y_{ih}^{cp} :=$
   40DWP produced from concentrated whey in plant $i \in V$ by configuration $h \in H_i^p$

Binary Variables

We have two types of binary variables

$$z^c_{ih} := \begin{cases} 1 & \text{If a collection center is located in node } i \in V \text{ using configuration } h \in H^c_i, \\ 0 & \text{Otherwise.} \end{cases}$$

$$z^p_{ih} := \begin{cases} 1 & \text{If a plant is located in node } i \in V \text{ using configuration } h \in H^p_i, \\ 0 & \text{Otherwise.} \end{cases}$$

Objective function

We maximize total profit

$$\begin{aligned}\max\ & s\sum_{i\in V}\sum_{h\in H_i^p}\left(y_{ih}^{rp}+y_{ih}^{cp}\right)\\ &-\sum_{i\in V}\sum_{h\in H_i^c}O_{ih}^{rc}\,y_{ih}^{rc}-\sum_{i\in V}\sum_{h\in H_i^p}\left(O_{ih}^{rp}\,y_{ih}^{rp}+O_{ih}^{cp}\,y_{ih}^{cp}\right)\\ &-\sum_{i\in V}\sum_{j\in V}T_{ij}^{c}\,x_{ij}^{cp}-\sum_{i\in N}\sum_{j\in V}T_{ij}^{r}\left(x_{ij}^{rc}+x_{ij}^{rp}\right)\end{aligned}\tag{1}$$

Constraints

At most one facility with only one configuration

$$\sum_{h \in H_j^c} z_{jh}^c + \sum_{h \in H_j^p} z_{jh}^p \leq 1 \qquad \forall j \in V \qquad (2)$$

Constraints . . .

Total installation cost must not exceed the given budget

$$\sum_{i \in V} \sum_{h \in H_i^c} F_{ih}^c \, z_{ih}^c + \sum_{i \in V} \sum_{h \in H_i^p} F_{ih}^p \, z_{ih}^p \leq B \tag{3}$$

Constraints . . .

Raw whey produced by all cheese makers must enter the supply chain

$$\sum_{i \in V} (x_{ji}^{rc} + x_{ji}^{rp}) = w_j \qquad \forall j \in N \qquad (4)$$

Constraints . . .

A known fraction of input raw whey is converted to concentrated whey in a collection center

$$\alpha \sum_{i \in N} x_{ij}^{rc} = \sum_{h \in H_j^c} y_{jh}^{rc} \qquad \forall j \in V \qquad (5)$$

Constraints . . .

A known fraction of input concentrated whey is converted to 40DWP in a plant

$$\beta \sum_{i \in V} x_{ij}^{cp} = \sum_{h \in H_j^p} y_{jh}^{cp} \qquad \forall j \in V \qquad (6)$$

Constraints . . .

A known fraction of input raw whey is converted to 40DWP in a plant

$$\alpha\beta \sum_{i \in N} x_{ij}^{rp} = \sum_{h \in H_j^p} y_{jh}^{rp} \qquad \forall j \in V \qquad (7)$$

Constraints . . .

Concentrated whey produced in a collection center must go to plants for further processing

$$\sum_{h \in H_j^c} y_{jh}^{rc} = \sum_{i \in V} x_{ji}^{cp} \qquad \forall j \in V \qquad (8)$$

Constraints . . .

Each configuration of a collection center allows us to produce at most a specific amount of concentrated whey

$$y_{jh}^{rc} \leq U_{jh}^{c} \, z_{jh}^{c} \qquad \forall j \in V, \, h \in H_j^c \tag{9}$$

Constraints . . .

Each configuration of a plant allows us to produce at most a specific amount of 40DWP

$$y_{jh}^{rp} + y_{jh}^{cp} \leq U_{jh}^{p} \, z_{jh}^{p} \qquad \forall j \in V,\, h \in H_j^p \qquad (10)$$

Constraints . . .

Concentration of raw whey will not take place in a collection center unless the required minimum capacity is met

$$y_{jh}^{rc} \geq L_j^c \, z_{jh}^c \qquad \forall j \in V,\, h \in H_j^c \qquad (11)$$

Constraints . . .

Production of 40DWP will not take place in a plant unless at least a specific amount of 40DWP is produced

$$y_{jh}^{rp} + y_{jh}^{cp} \geq L_j^p \, z_{jh}^p \qquad \forall j \in V, \, h \in H_j^p \qquad (12)$$

Constraints . . .

Domain of variables

$$z_{jh}^{c} \in \{0,1\} \qquad \forall j \in V,\ h \in H_j^c \tag{13}$$

$$z_{jh}^{p} \in \{0,1\} \qquad \forall j \in V,\ h \in H_j^p \tag{14}$$

$$y_{jh}^{rc} \geq 0 \qquad \forall j \in V,\ h \in H_j^c \tag{15}$$

$$y_{jh}^{rp} \geq 0 \qquad \forall j \in V,\ h \in H_j^p \tag{16}$$

$$y_{jh}^{cp} \geq 0 \qquad \forall j \in V,\ h \in H_j^p \tag{17}$$

$$x_{ij}^{rc} \geq 0 \qquad \forall i \in N,\ j \in V \tag{18}$$

$$x_{ij}^{rp} \geq 0 \qquad \forall i \in N,\ j \in V \tag{19}$$

$$x_{ij}^{cp} \geq 0 \qquad \forall i \in V,\ j \in V \tag{20}$$

How to solve it?

- It is a mixed integer **linear** program
- We just need to use a state-of-the-art linear solver

- Real world case (see reference):
  - The actual whey supply chain model has 52 nodes with 10 candidate centers.
  - It has been solved using both LP Solve 5.5.2 and CPLEX 12.5 in around 2 minutes.

Outline

Consideration of uncertainty

- Uncertainty in which parameters?
  - Budget available for construction,
  - The transportation costs,
  - Selling price of our commercial product(s)
  - The availability of configurations,
  - ...
- Which approach to take?
  - Stochastic Programming
  - Robust Optimization

Reference and contact information

Rodolfo Garcia-Flores, Ricardo Martins, Osmar Vieira de Souza Filho, Marcelo Gonzalez, Carlos Mattos, Amauri Rosenthal and Pablo Juliano

A novel facility and equipment selection model for whey utilisation: A Brazilian case study, Computers and Electronics in Agriculture. 2015 (117), 127 – 140.

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Thank You