Biarri
COMMERCIAL MATHEMATICS

SCHEDULING OF MEDICAL PROCEDURES

WHY TO PERSEVERE WITH A MIP APPROACH

BAM 2014 – DR MICHAEL FORBES (AND STUDENTS)

AGENDA
Why we give up on a MIP approach
Why we should persevere
Scheduling of medical procedures
Questions

Why we give up on a MIP approach to optimisation problems

- Our problem is not linear
- MIP is too slow
- MIP run times can blow up with small data changes
- MIP run times can blow up with small model changes
- Good MIP solvers cost money

- Good MIP modellers cost money

Why we should persevere with a MIP approach to optimisation problems

- You get a bound on solution quality
- It makes you think about modelling the problem
- Someone else is dedicated to making your problem run faster
  - Gurobi ~ 30 times speedup (software only) since Biarri started
  - 5.63 to 6.0
    - Easy (90 problems): 5.7 sec to 5.3 sec;
    - Medium (40 problems): 209.9 sec to 140.0 sec;
    - Unsolved in 1 hour (50 problems): 0.67% gap to 0.49%
    - One additional hard problem solved
- Many ways to make MIPs faster
- Many MIP based heuristics.

Making your MIP faster – Tighter formulation

$$\min \sum_{i,j=1..N} d_{ij} x_{ij}$$

Subject to:

$$\sum_{i=1..N} x_{ij} \leq N y_j \quad \forall\, j$$

$$\sum_{j} x_{ij} = 1 \quad \forall\ i$$

$$\sum_{j} y_j \leq M$$

$$x_{ij}, y_j \in \{0,1\}$$

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$$\sum_{j} y_j \leq M$$

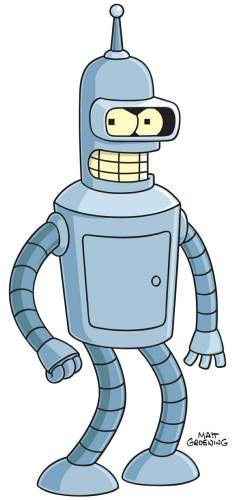
$$x_{ij}, y_j \in \{0,1\}$$

Making your MIP faster

- Composite variables:
  - Tighter formulation
  - Reduce symmetry
  - Embodies difficult constraints (e.g. crew scheduling)
  - May be the only way to model the problem
- Delayed column generation / Dantzig-Wolfe decomposition / Branch & Price
  - Useful when there are an exponential number of composite variables
  - Col gen done with CP, DP, MIP
- Keeping track of all columns “generated” by a heuristic search procedure
  - Very useful for VRP and variants.

Making your MIP faster

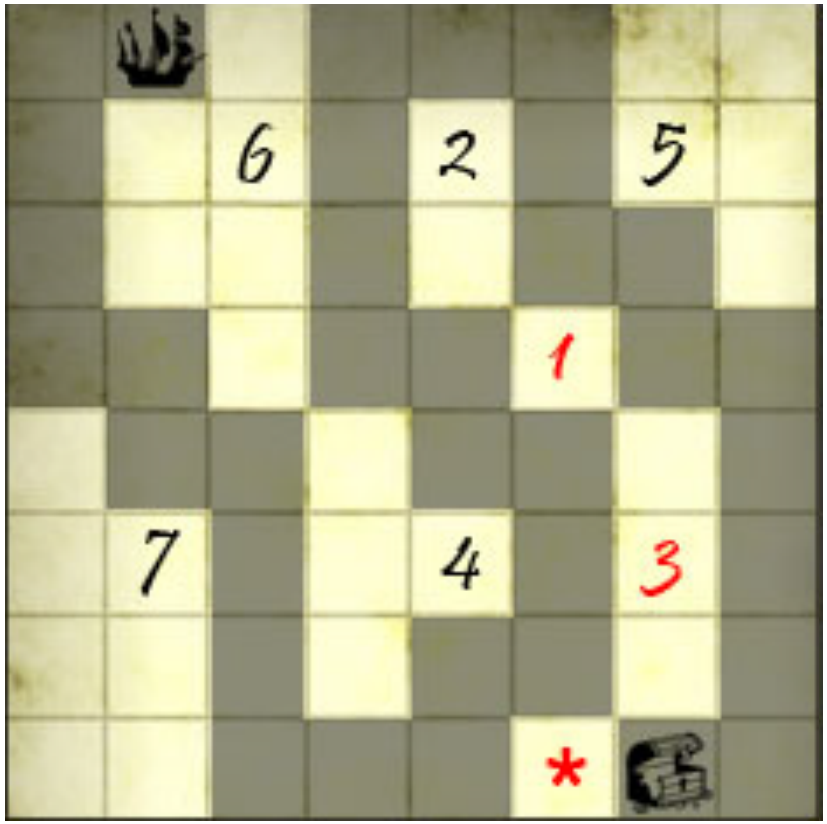
- Benders' Decomposition
  - Disaggregated cuts
  - Added in callback
  - Integer "structural" variables
  - Classic way to solve Stochastic MIPs



- Lagrangian relaxation
  - Relaxed problem must be "not too easy" and "not too hard"

Making your MIP faster

- Lazy Constraints
- Pieces of 8
  - $x_{ijk} = 1$ if square (i,j) is type k
  - Each square is used once
  - Squares of type 0 have exactly 2 neighbours of type 0 (except origin and destination have 1)
  - The right number of squares for each piece of 8
  - Pieces of different types aren't neighbours
  - At least one neighbour of the same type
- Plus …
  - Each piece of 8 is connected
  - There are no loops in the path.



Making your MIP faster

- Tighter formulation
- Composite variables / Delayed Column Generation
- Benders Decomposition
- Lagrangian Relaxation
- Lazy Constraints

Scheduling Medical Procedures

- Treatment – a list of procedures for a patient
- Procedure
  - Duration
  - List of suitable equipment
- Equipment
  - Specified maximum simultaneous patient capacity
  - List of eligible supervising physicians
  - Is it critical?

1) Multi-objective scheduling and a resource allocation problem in hospitals, Silvija Vlah Jeri´c and José Rui Figueira, J Sched (2012) 15:513–535
2) Work done by Sean Watson

Scheduling Medical Procedures

- Objectives:
  - Maximise number of treatments performed
  - Maximise the times when critical equipment is free
  - Minimise the waiting time of physicians
- Subject to:
  - The procedures in each treatment are completed in the correct order without gaps
  - All procedures are performed on the correct equipment and with an eligible physician.
  - Multiple physicians and patients may use the same piece of equipment at any time as long as capacities are complied with.
  - Each procedure must be performed from start to finish on the same piece of equipment and with the same physician.
  - Physicians can only supervise one piece of equipment at a time

Scheduling Medical Procedures

IP Formulation as presented in paper	
x_{it}	1 is treatment i starts at time t
y_{ilt}	1 if treatment i is being performed on equipment l at time t
z_{hlt}	1 if physician h is supervising equipment l at time t
w_{ijh}	1 if procedure j of treatment i is supervised by physician h

Improved IP Formulation	
x_{it}	1 is treatment i starts at time t
y_{ijhlt}	1 if procedure j of treatment i starts at time t on equipment l, supervised by physician h
z_{hlt}	1 if physician h supervises equipment l at time t

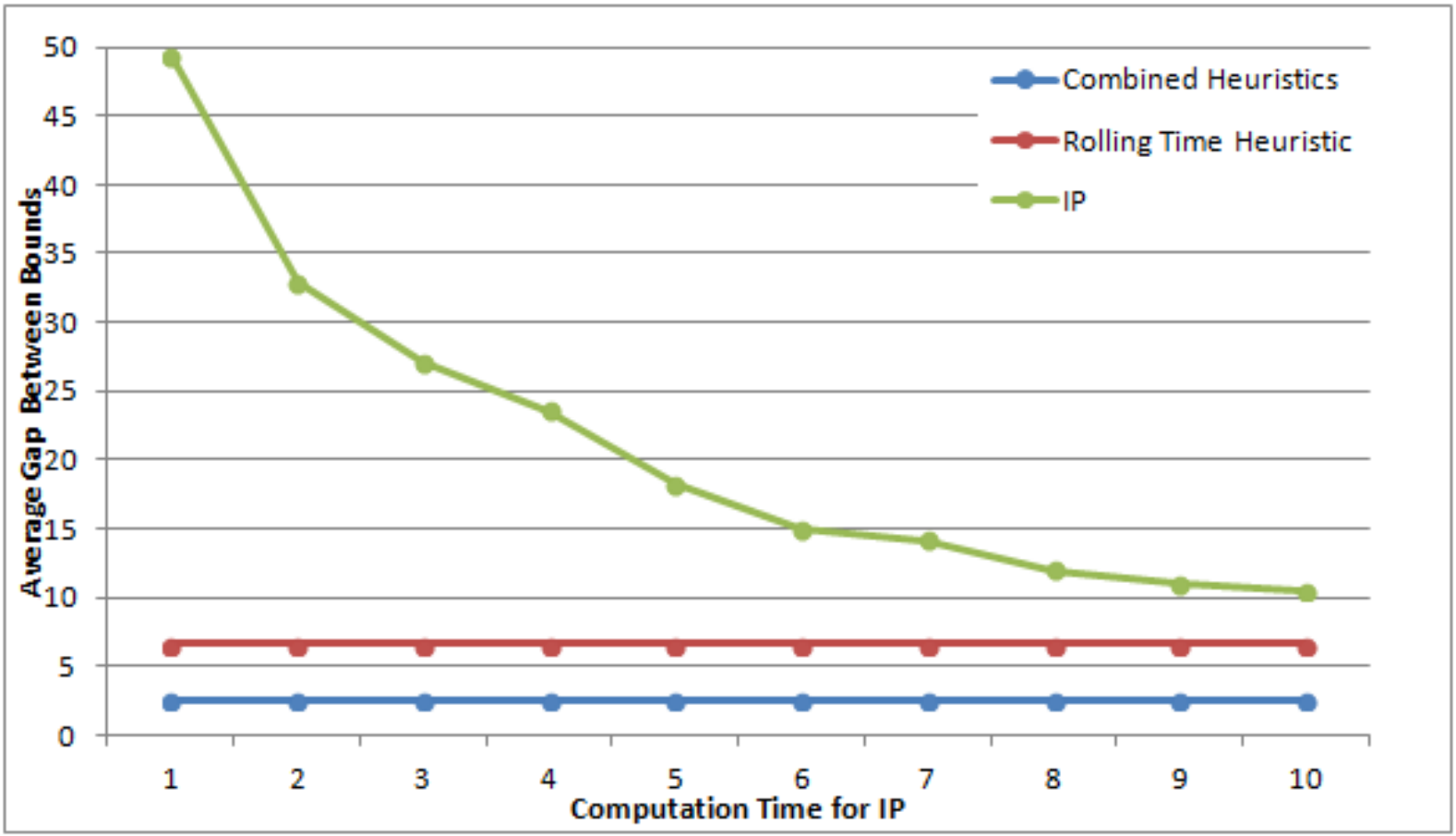
Results

Problem		Constraints		Variables		Time (s)		Gap %	
I	T	Orig	Impr	Orig	Impr	Orig	Impr	Orig	Impr
5	15	1857	591	2437	916	8.69	0.63	0	0
7	20	3701	2761	3711	1588	23.35	0.53	0	0
9	20	4955	1441	4412	2994	428.75	2.94	0	0
11	25	7007	1967	7259	3829		2.87	4.00	0
13	25	8188	2186	8413	4730		3.28	1.64	0
15	25	8846	2385	6917	5315		27.41	17.21	0
17	25	9337	2183	9626	5020		218.83	8.16	0
19	25	10451	2423	10035	5723			15.38	1.71
25	25	13293	2823	10567	7705			26.98	10.95
30	30	19705	4129	13420	11427			58.70	10.74

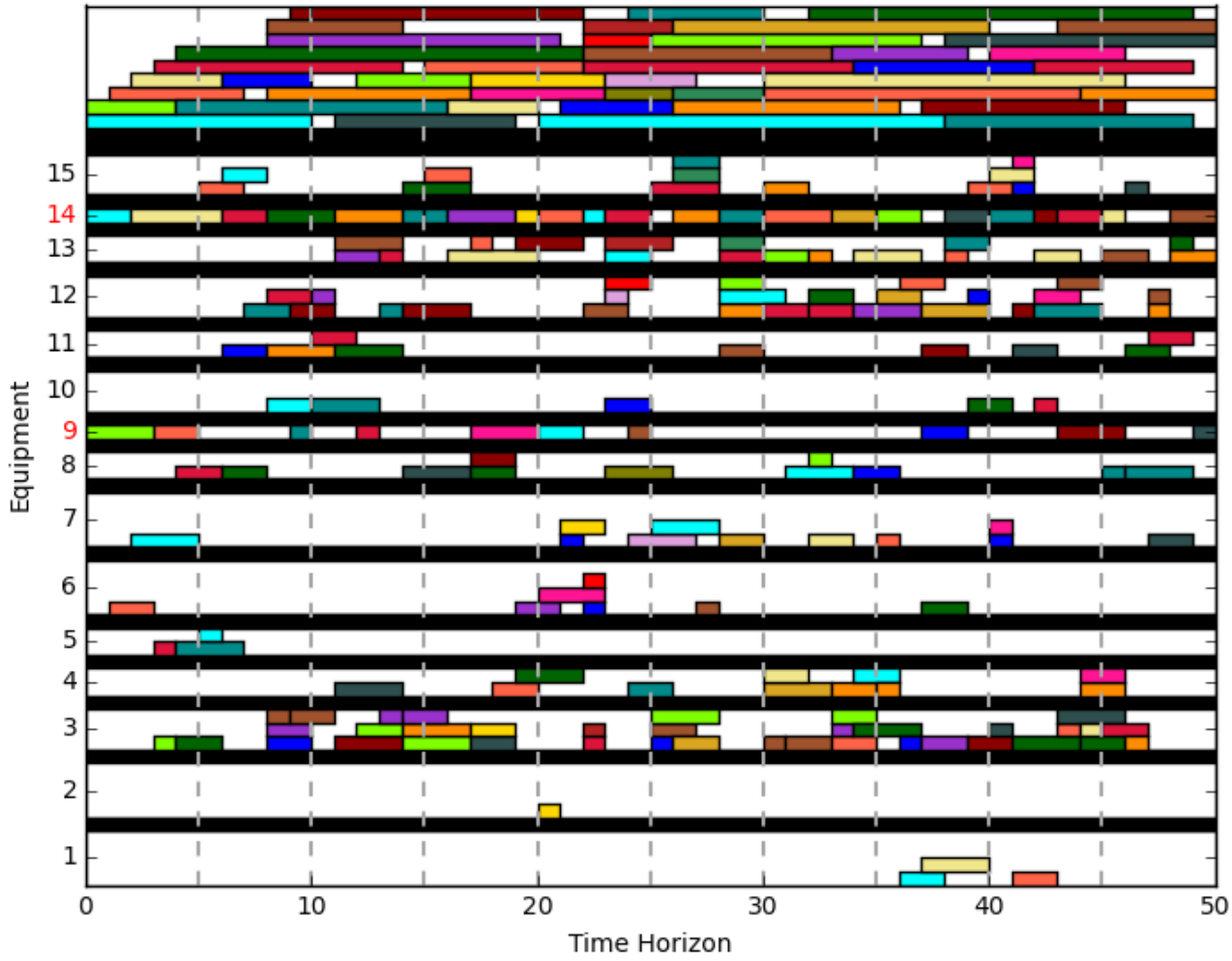
IP Heuristics

- Rolling time
  - Suitable for problems with an emerging time horizon
  - Solve rolling time bands to integrality, with overlap
- Fix and flex style heuristics
  - Fix part of a known solution and optimise the rest

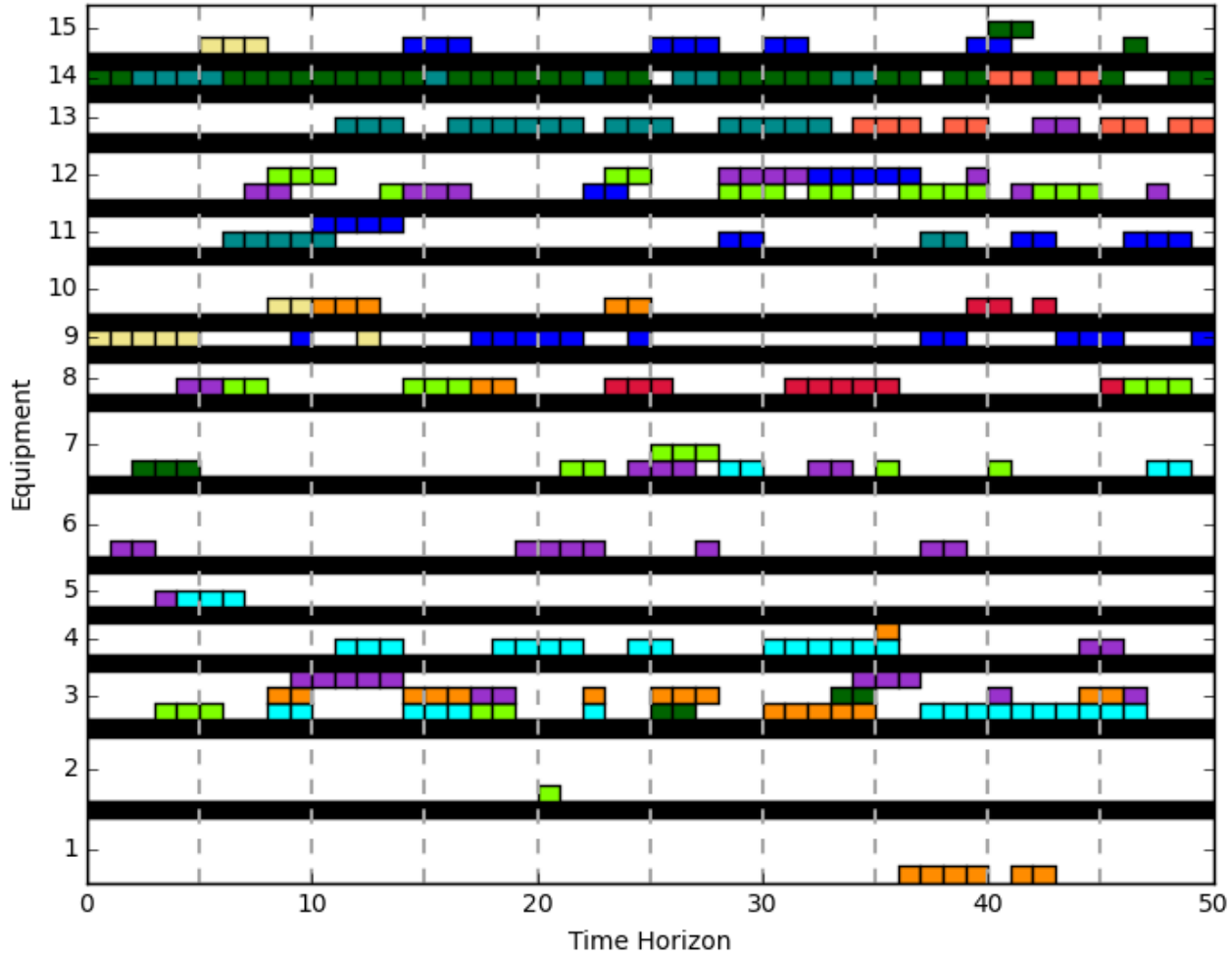
Results


Combined Heuristics
Rolling Time Heuristic
IP
Average Gap Between Bounds
50
45
40
35
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
10
Computation Time for IP

Treatments on Equipment



Physicians on Equipment



Conclusions

- More and more problems can be handled by MIP
- Having a quality guarantee (a bound) is priceless
- Always try MIP first
- Don’t give up too quickly
- Familiarise yourself with the state of the art

- Questions?