Traffic modelling, from fundamentals to large network simulations

Jan de Gier

ACEMS, University of Melbourne

BAM Conference, RMIT 2014

Outline

1. Cellular Automata
   - M/M/1 Queue
   - The asymmetric exclusion process
   - Nagel-Schreckenberg process

2. Two-dimensional models
   - NetNasch
   - Open problems

Collaborators

- Tim Garoni (Monash University)
- **Lele (Joyce) Zhang** (Monash University)
- Somayeh Shiri (Monash University)
- Omar Rojas (La Trobe University)
- Andrea Bedini (University of Melbourne)
- Caley Finn (University of Melbourne)
- John Foxcroft (University of Melbourne)
- ...
- **VicRoads**

History

- (2008) ARC Centre of Excellence MASCOS begin working with VicRoads
  - Six-month AMSI Industry Internship with VicRoads

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- (2015 -) ARC CoE ACEMS/VicRoads develop real world applications
  - NetNaSch, a versatile model for road traffic networks
  - CEllular Automaton Simulator for Arterial Roads (CEASAR)
  - Inner North Road and Tram Network
  - VicRoads Policy Development

M/M/1 Queue

Arrival rate α
Service rate β

M/M/1 Queue

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Length probability distribution function P_n obeys master equation

$$\tilde{P}_0 = \beta P_1 + (1-\alpha) P_0$$
$$\tilde{P}_n = \alpha P_{n-1} + \beta P_{n+1} + (1-\alpha-\beta) P_n \qquad n > 0.$$

Factorised stationary solution:

$$P_n = \left(1 - \frac{\alpha}{\beta}\right)\left(\frac{\alpha}{\beta}\right)^n \qquad \alpha < \beta.$$

Asymmetric simple exclusion process (ASEP)

- One-dimensional stochastic cellular automata very popular in statistical mechanics starting in 1990s
- Such models do a reasonable job of explaining qualitative behaviour of freeway traffic
- Sponaneous jams emerge as consequence of collective behaviour
- Advanced analytical methods (integrability, random matrix theory)

- If $x_1(t) = 0$, then with probability α, $x_1(t+1) = 1$
- If $x_L(t) = 1$, then with probability β, $x_L(t+1) = 0$
- If $x_i(t) = 1$ and $x_{i+1}(t) = 0$ then with probability p, $x_i(t+1) = 0$ and $x_{i+1}(t) = 1$

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Asymmetric simple exclusion process (ASEP)

Stationary state no longer simply factorised

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but can use **matrix** product formalism

$$P_L(x_1, \ldots, x_L) = \frac{1}{Z_L} \langle W| \prod_{i=1}^{L} [x_i D + (1 - x_i) E] \, |V\rangle,$$

with normalisation

$$Z_L = \langle W|(D+E)^L|V\rangle = \langle W|C^N|V\rangle, \qquad C = D + E$$

For example,

$$P_5(10110) = \frac{1}{Z_5} \langle W|DEDDE|V\rangle$$

If matrices are known, it is easy to compute quantities of interest.

- Density:

$$\rho_i = \mathbb{E}[x_i] = \frac{1}{Z_N} \langle W|C^{i-1} D C^{N-i}|V\rangle$$

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- Current:

$$J_{i,i+1} = p\,\mathbb{E}[\tau_i(1-\tau_{i+1})] = \frac{p}{Z_N} \langle W|C^{i-1}DEC^{N-i-1}|V\rangle$$

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The matrices satisfy the relations:

$$\begin{aligned} pDE &= D + E, \\ \langle W | E &= \frac{1}{\alpha} \langle W |, \\ D | V \rangle &= \frac{1}{\beta} | V \rangle. \end{aligned}$$

Hence

$$J_{i,i+1} = J = \frac{p}{Z_N} \langle W | C^{i-1} D E C^{N-i-1} | V \rangle = \frac{Z_{N-1}}{Z_N}$$

Explicit matrix representation

$$D = \frac{1}{p}\begin{pmatrix} 1 & 1 & 0 & 0 & \cdots \\ 0 & 1 & 1 & 0 & \\ 0 & 0 & 1 & 1 & \\ 0 & 0 & 0 & 1 & \\ \vdots & & & & \ddots \end{pmatrix}, \qquad E = \frac{1}{p}\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots \\ 1 & 1 & 0 & 0 & \\ 0 & 1 & 1 & 0 & \\ 0 & 0 & 1 & 0 & \\ \vdots & & & & \ddots \end{pmatrix}$$

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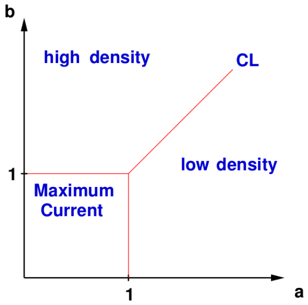
$$\langle W| = \frac{\kappa}{p}(1, a, a^2, a^3, \ldots), \qquad |V\rangle = \frac{\kappa}{p}(1, b, b^2, b^3, \ldots)^T$$

where

$$a = \frac{1-\alpha}{\alpha}, \qquad b = \frac{1-\beta}{\beta}, \qquad \kappa = \frac{1}{\alpha} + \frac{1}{\beta} - \frac{1}{\alpha\beta}.$$

ASEP Phase diagram



Analytic expressions for

- Stationary state
- Density and current profiles
- Relaxation rates
- Fluctuations, large deviations

Mathematical techniques

- Stationary state: Matrix product states
- Relaxation rates: Eigenvalues of transition matrix $\Rightarrow$ Integrability
- Large deviations, current generating functions: Random matrix techniques, Integrability

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Example: **Current large deviation function**
Let $Q_1(t)$ be the total time-integrated current, i.e., the net number of particle jumps between the left boundary reservoir and site 1 in the time interval $[0, t]$.

Moments are encoded in $\langle \mathrm{e}^{\lambda Q_1(t)} \rangle$ (average over histories)

Theorem (Current fluctuations)

In the low density regime

$$E(\lambda) := \lim_{N\to\infty} \lim_{t\to\infty} \log \frac{1}{t} \langle \mathrm{e}^{\lambda Q_1(t)} \rangle = p \frac{a(\mathrm{e}^{\lambda} - 1)}{(1+a)(\mathrm{e}^{\lambda} + a)}.$$

Nagel-Schreckenberg process

- Generalises ASEP
  - Vehicles can have different speeds $0, 1, \ldots, v_{\max}$

- x_n and v_n denote position and speed of the nth vehicle
- d_n denotes the gap in front of the nth vehicle
- NaSch rules:
  - $v_n \mapsto \min(v_n + 1, v_{\max})$
  - $v_n \mapsto \min(v_n, d_n)$
  - $v_n \mapsto \max(v_n - 1, 0)$ with probability p
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Traffic modelling: network of cellular automata

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A comparative study of Macroscopic Fundamental Diagrams of arterial road networks governed by adaptive traffic signal systems

CrossM

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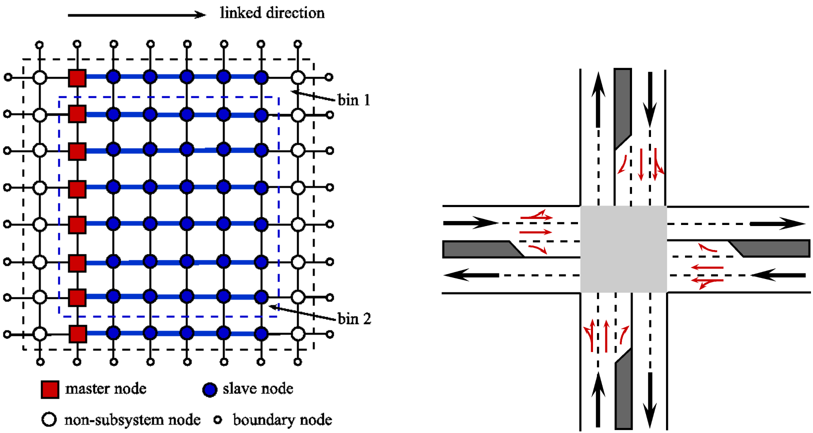
Traffic modelling: network of cellular automata



(a) Road network (b) Intersection

NetNaSch: A mesoscopic model of arterial networks

- Ingredients:
  - Dynamics on lanes modelled with (extended) NaSch model
  - Multiple lanes with lane changing
  - Turning decisions (random)
  - Appropriate rules for how vehicles traverse intersections
    - Sydney Coordinated Adaptive Traffic System (SCATS)

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- Features
  - Multiple agent types: cars, trams, buses, bicycles, pedestrians, ...
  - Easily applied to arbitrary network structures
  - Essentially any rules for traffic signals can be implemented
  - Relatively modest amount of input data required

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  - Relatively modest amount of input data required
- Goals:
  - Statistical mechanics of traffic (phase tranistions, non-equilibrium behaviour)
  - New scenario analysis, generic behaviour
  - Policy development
    - Parking restrictions
    - Change signal systems
    - Tram priority

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11/24/2014 CEASAR - Cellular Automata Simulator for Arterial Roads

http://www.ms.unimelb.edu.au/~degier/ACEMS/CEASAR/ 2/4

http://ceasar.acems.org.au

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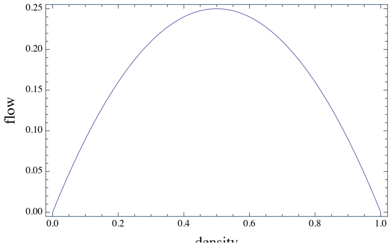
Fundamental Diagram

- The functional relationship between flow and density is the **fundamental diagram**



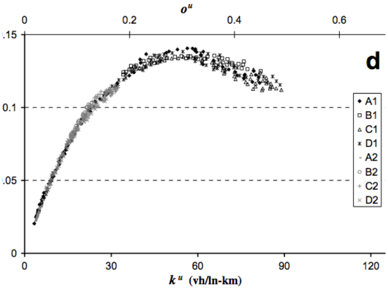
- Can be computed analytically for ASEP
- What should happen in a network?
- How should one even **define** network flow?
  (No prescribed direction)

Macroscopic Fundamental Diagrams

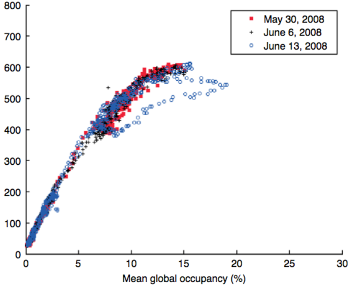
- Simplest idea: relate arithmetic means of link density and flow
- If network has link set Λ:

$$\rho = \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}\rho_\lambda, \qquad J = \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}J_\lambda$$

- ρ_λ is density of link λ and J_λ is its flow



Geroliminis & Daganzo 2008
Empirical data from Yokohama



Buisson & Ladier 2009
Empirical data from Toulouse

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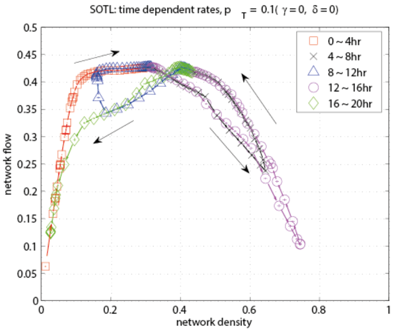
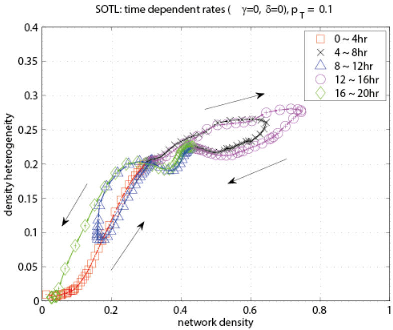
Time-dependent demand

- Hysteresis in MFD consequence of heterogeneity



Zhang, G & de Gier 2013

Simulated data

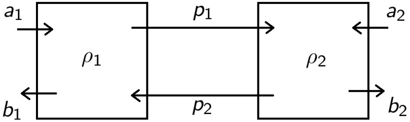


Zhang, G & de Gier 2013

Simulated data

Two-bin model

Can a simple model explain hysteresis?



$$\frac{d\rho_1}{dt} = A(\rho_1) - b_1 J(\rho_1) + J(\rho_2) p_2 (1 - \rho_1) - J(\rho_1) p_1 (1 - \rho_2),$$
$$\frac{d\rho_2}{dt} = A(\rho_2) - b_2 J(\rho_2) - J(\rho_1) p_1 (1 - \rho_2) + J(\rho_1) p_2 (1 - \rho_1)$$

Let bin 1 be the boundary and bin 2 the interior.
Loading and recovery phases of the two-bin model provide explanation

Two-bin model

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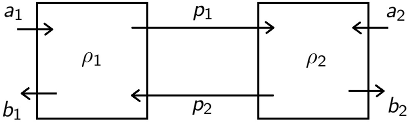

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Theorem (Law of traffic)

It is easier to jam a network than to resolve it.

Open problems

- Good theoretical models amenable to non-numerical analysis
- NetNaSch: A realistic and flexible model of traffic in arterial networks
- CEASAR: browser based CEllular Automata Simulator for Arterial Roads
- Real time estimators of network behaviour
- How does driver adaptivity affect the shape of MFDs?
- Study on tram priority in Melbourne
- **Big data**: Integrate stopline data (density), GPS data of probe vehicles and predictive modelling