Wolf - Lamb Cutting Optimisation Tool

(Silence of the Lambs)

Tech Talk

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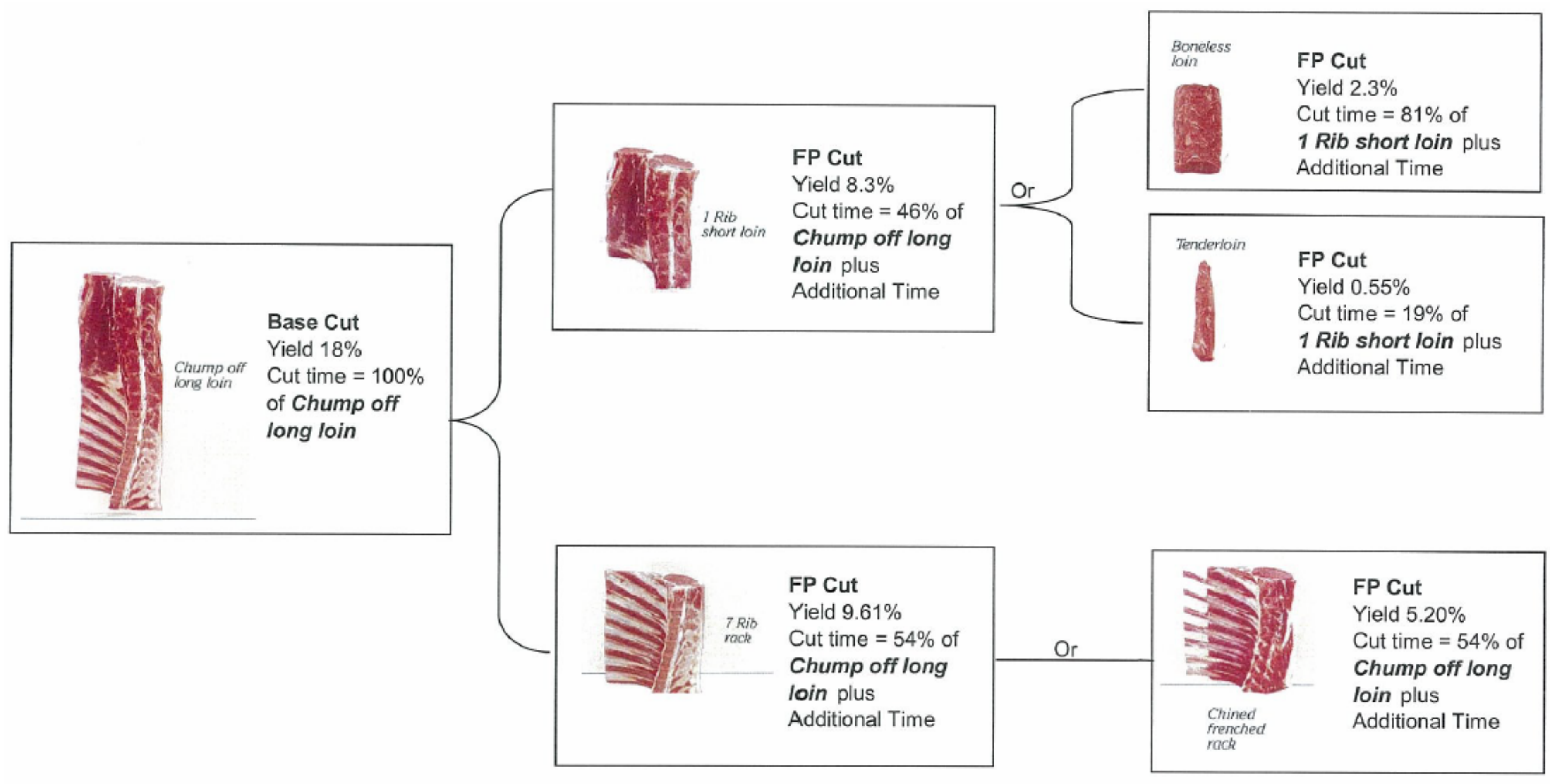
COMMERCIAL MATHEMATICS

Maximise revenue for world's largest exporter of lamb and mutton products

- Produces mutton and lamb products from millions of carcasses per annum (10's of thousands slaughtered in a day!)
- Over half of the world's export market
- There are many ways in which carcases can be divided into cuts, while a particular cut may be sold as a number of different items (e.g. chilled or frozen)
- When demand is higher than supply it is non-trivial to determine which order mix maximises revenue.
- Revenue calculated from:
  - Orders
  - Sale price
  - Labour, packaging, shipping, finance costs

The same part of the carcass can be cut in many different combinations to create a variety of different products

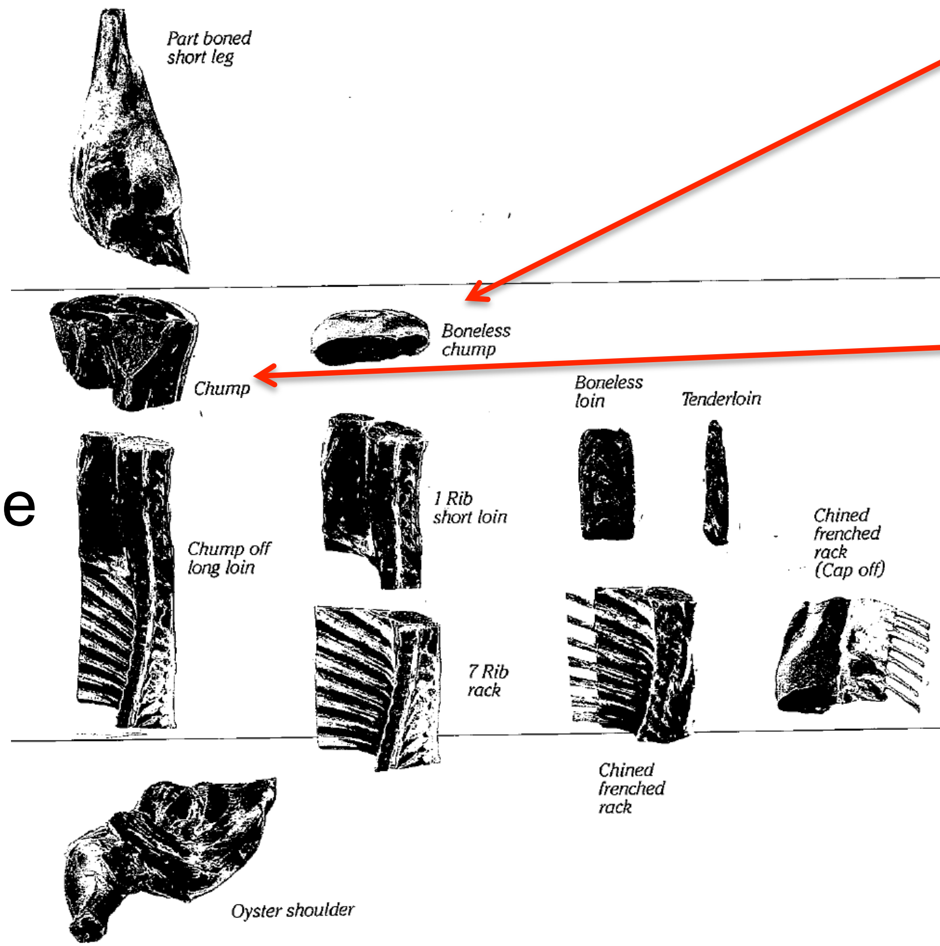


More product combinations

Leg

Middle

Fore



Boneless chump has:
- lower yield
- higher production cost

But
- Higher revenue per kg when compared to a normal chump

Primal Types: Leg, Middle, Fore

Determine the optimum production mix (quantity by item) given production process rules and throughput constraints

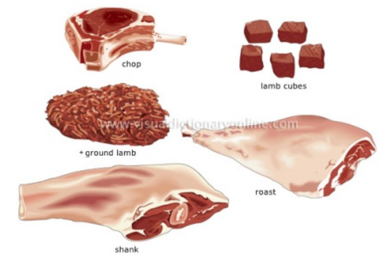
Input

- Kill Table: species, plant, grade, number, average weight
- Product Table: species, product, yield, VRA Factor, Chump flag
- Cut details: yields, VRA coefficients
- Product Combinations Table
- Stpk Table: product, market, state (chilled, fresh), costs, cut time, currency, CIF, Link, Tonnes, Tolerance (min%, max %), Grades, Plant
- Currency Rates
- VRA Market Table

Engine

- Select production mix to maximise implied revenue
- Obey market, production and loss leader related constraints

Output



- Optimum production mix (quantity by stpk item)
- Associated cutting specifications
- Implied profit by stpk item and by market
- Unused carcass supply, unmet market demand, unsold ("stored") production

Primal Type Combinations

Name	Type 1	Type 2	Type 3	Type 4
Carcass	Carcass			
6WayCut	6WayCut			
1	Leg	Middle	Fore	Chump
2	Leg	ExtMiddle	ShortFore	Chump

Some numbers

- Species: 2 (Lamb, Mutton)
- Plants: < 10 around country
- Carcasses: (over 1M/year)
- Orders: several 100

- Grades: < 20 (Lamb and Mutton)
- Products: < 100 (Lamb and Mutton)
- Product Combinations:

	Lamb	Mutton
Leg	< 20	< 10
Middle	< 40	< 20
Fore	< 30	< 10

Order/Demand Details

- Customer Name
- Species (e.g. Lamb, Mutton)
- Product (e.g. 7Ribrack)
- Packaging Cost per kg (e.g. 0.10)
- Freight Cost per kg (e.g. 0.70)
- Cut time (e.g. 40 seconds)
- Weeks (e.g. 4)
- Currency (e.g. EUR, USD, CAD ..)
- CIF_1 (cost in foreign currency) (e.g. 5.60)
- CIF_2 (e.g. 5.40)
- Tonnes (e.g. 50)
- CIF threshold (e.g. 40)
- Min Tonne (e.g. 45)
- Min Tonne % (e.g. 10%)
- Max Tonne % (e.g. 10%)
- Grade Options (e.g. Pm(100%), px(50%))
- Plant Options (e.g. Lnv(100%), puk(50%))

Outputs

- Satisfied Demand
- Supply Excess
- Demand Shortfall
- KPIs

KPI	1000's
Total Revenue	Lots (100's millions)
Total Freight Cost	Small (10's millions)
Total Cut Cost	Small (10's millions)
Total Packaging Cost	Small (10's millions)
Total Finance Cost	Medium (over 100 million)
Net Revenue	Total Revenue - Costs
Total Supplied Tonnes	Over 100k
Demand Shortfall Tonnes	Zero (for a balanced model)
Total Carcass Input Tonnes	Total Supplied Tonnes – Off cuts

Maths

1 + 1 = ?

a) 2
b) 11
c) A Window
d) All of the above
e) None of the above
f) Some of the above

Maximise Objective Function

$$\sum_{d\in D} (R_{dCIF1}\, x_{dCIF1} + R_{dCIF2}\, x_{dCIF2})$$
(revenue)

$$-\sum_{d\in D, g\in G, l\in L} C_{dg}\, x_{dgl}$$ (labour cost)

$$-FR * \sum_{d\in D} Weeks_d / 52\, (R_{dCIF1}\, x_{dCIF1} + R_{dCIF2}\, x_{dCIF2})$$ (finance cost)

$$-\sum_{d\in D, g\in G, l\in L} O_d\, x_{dgl}$$ (packaging and freight cost)

$$-\sum_{d\in D} q_d\, s_d$$ (unsatisfied demand cost)

Let $x_{dCIFx} \in R$ be the amount of demand d satisfied at CIFx price in kg

Let $x_{dgl} \in R$ be the amount of demand d satisfied for grade g from plant l in kg

Let $s_d \in R$ be the amount of unsatisfied demand d

Let R_{dCIFx} be the revenue associated with 1kg of demand d at CIFx

Let FR = Finance Rate

Let O_d be the 'other' costs per kg for demand d (sum Packing and Freight cost)

Let q_d be the penalty/cost applied to unmet demand for demand d

Constraints

1. Satisfy Minimum Demand (soft constraint)

   $\sum_{g \in G, l \in L} x_{dgl} + s_d = MIN_d \quad \forall\ d \in D$ (s_d = unsatisfied demand for demand d)

2. Don't exceed the maximum demand

   $\sum_{g \in G, l \in L} x_{dgl} \leq MAX_d \quad \forall\ d \in D$

3. Demand satisfied = demand CIF1 + CIF2

   $x_{dCIF1} + x_{dCIF2} = \sum_{g \in G, l \in L} x_{dgl} \quad \forall\ d \in D$

Constraints (cont)

4. Don't exceed maximum threshold

   $x_{dCIF1} \le CT_d$ $\forall\, d \in D$ (CT_d = CIF threshold for demand d)

5. Control how much of one grade can be used for d

   $\sum_{l \in L} x_{dgl} \le MAX_{dg}\,(x_{dCIF1} + x_{dCIF2})$ $\forall\, d \in D, \forall\, g \in G$ (MAX_{dg} = max amount of demand d for grade g)

6. Control how much of one plant can be used for d

   $\sum_{g \in G} x_{dgl} \le MAX_{d\square}\,(x_{dCIF1} + x_{dCIF2})$ $\forall\, d \in D, \forall\, l \in L$ ($MAX_{d\square}$ = max amount of demand d for plant l)

7. Comply with VRA limits

   $\sum_{g \in G,\, l \in L,\, d \in D_{VRA}} x_{dgl}\, VRA_d \le VRA_{max}$ (VRA_d is a demand factor for demand d)

Integer Constraints

e.g. leg, middle, fore

8. Primal combinations used≤total carcasses

$$\sum_{j\in J} z_{jgl} \le N_{gl} \quad \forall\, g\in G, \forall\, l\in L$$

e.g. product combinations that can come from a leg

$N_{gl} \in I$: number of carcasses of grade g at plant l

$z_{jgl} \in I$: number of times primal combination j has been used for grade g at plant l

9. Num of primal types q for comb j= primal combinations

$$a_{qjgl} = I_{qj}\, z_{jgl} \quad \forall\, q\in Q, \forall\, j\in J, \forall\, g\in G, \forall\, l\in L$$

$a_{qjgl} \in I$: number of times primal type q has been used from primal combination j for grade g and plant l

$I_{qj} \in \{0,1\}$: 1 if primal type q is in primal combination j, otherwise 0

$z_{jgl} \in I$: number of times primal combination j has been used for grade g at plant l

Integer Constraints (cont)

i.e. sum over product combinations j from previous constraint

10. Number of primal type q used for grade g at plant l

$$\sum_{j \in J} a_{qjgl} = a_{qgl} \qquad \forall\, q \in Q,\ \forall\, g \in G, \forall\, l \in L$$

11. Product combinations used for primal type=used primal types

$$\sum_{k \in K} V_{kq}\, e_{qkgl} = a_{qgl} \qquad \forall\, q \in Q,\ \forall\, g \in G, \forall\, l \in L$$

$V_{kq} \in \{0,1\}$: 1 if product combination k uses primal type q, otherwise 0

$e_{qkgl} \in I$: number of times product combination k for primal type q has been used for grade g at plant l

Integer Constraints (cont)

12. Number of product p of grade g at plant l

$$\sum_{k \in K} H_{pk} e_{qkgl} = b_{pgl} \qquad \forall p \in P, \forall g \in G, \forall l \in L$$

$H_{pk} \in \{0,1\}$: 1 if product p is in product combination k, otherwise 0

13. Demand satisfied $\leq$ *total yield from products created*

$$\sum_{d \in D_p} x_{dgl} \leq b_{pgl} W_{gl} Y_p \qquad \forall p \in P, \forall g \in G, \forall l \in L$$

Total weight of product p at plant l of grade g

$b_{pgl} \in I$: number of times product p has been used for grade g at plant l

W_{gl} : average weight of grade g at plant l.

Y_p : yield for product p (% of entire carcass)

$D_p \subset D$: set of demands for product p

What's next

- Client has shown interest in using the tool for other products (they also deal with beef and venison). The tool that we've put together can be easily adapted.
- Several other areas within their business that we believe we can provide some value (e.g. route optimisation)
- Plans to sell the tool (and a number of variants) to other clients